homework 3

cs201 **due 10 february 1999**

quiz problems

1. Give a formal deduction (use the rules of logic in a numbered list) of the following theorems:

a)

$$\frac{\neg p \to F}{\therefore p}$$

b)

$$\frac{\neg q \to \neg p}{\therefore p \to q}$$

c)

$$\begin{array}{l} p_1 \vee p_2 \\ (p_1 \to q) \wedge (p_2 \to q) \\ \hline \therefore q \end{array}$$

d)

$$\begin{array}{l} p \to q \\ q \to r \\ r \to p \\ \hline \therefore p \leftrightarrow r \end{array}$$

2. Translate the following sentences into predicate form. Use the predicate $L(x, y)$ to mean x loves y.

   a) I love everyone.
   b) Nobody loves me.
   c) Everyone has someone who loves them.
   d) No one loves everyone.

3. Prove that $m + n$ is even if and only if n and m are both even or n and m are both odd.

4. Let x, y, and z be real numbers. Let a be their average. Prove that at least one of x, y, or z is equal to or greater than a.

5. Let $P(n)$ be $n^3 + 2n$ is divisible by 3. Prove that $P(x)$ is true for all $x \geq 1$.

6. A finite collection of *squares* (possibly overlapping) in a plane cuts the plane into finitely many regions. Prove or disprove: these regions can be colored with only two colors such that no two adjacent regions are of the same color.

real problems

1. Let $P(n)$ be $\sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}$. Prove that $P(x)$ is true for all $x \geq 1$.

2. Let $P(n)$ be $(a + b)^n \geq a^n + b^n$ for positive values of a and b. Prove that $P(x)$ is true for all $x \geq 1$.

3. I have a 4 oz. measuring cup and 5 oz. measuring cup. Using them, I can't measure 1, 2, 3, 6, 7, or 11 oz., but I can measure any other whole-numbered size. Prove I'm right.

4. A finite collection of *circles* (possibly overlapping) in a plane cuts the plane into finitely many regions. Prove or disprove: these regions can be colored with only two colors such that no two adjacent regions are of the same color.

extra credit

Prove the circle problem above without using mathematical induction.